INFLUENCE OF NOISE ON THE QUASI-HOMOCLINIC BEHAVIOR OF A LASER WITH SATURABLE ABSORBER

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The influence of noise on a laser with saturable absorber is investigated. In particular, the effect of an external added noise on the return time fluctuations and on the hesitation regime is studied. Numerical simulations based on a model with three-levels in the amplifier and four-levels in the absorber lead to an interpretation of these behaviors.

1. Introduction

The laser containing an intracavity saturable absorber (LSA) is a system studied since the beginning of the quantum optics. This system presents two characteristic nonlinear behaviors, optical bistability and a periodic pulsed regime known as Passive Q-Switch (PQS). Extensive studies have been made on LSA during the last years in the context of optical computing, because this system appears well apt to characterize bistable cycles [1]. More recently, the PQS studies have been resumed through the developments of the instability interpretations [2]. In particular, a systematic exploration of LSA systems presenting PQS has led to the discovery of new PQS regimes, with links to different kinds of chaos [2–5], and to the elaboration of new models [4, 6–8]. The relevant classification of the PQS regimes, resulting from the experimental observations, is as in the following:

(i) For a large amplifier gain and/or a weak absorber appears the so-called type II-PQS [2], constituted in his simplest case of a periodic modulation of the laser output power at a frequency in the 50–100 kHz range. This regime may evolve to a chaotic one through a periodic doubling cascade [2,3].

(ii) In the PQS regime called $P^{(n)}$, each periodic laser output pulse is composed by an intense peak followed by a series of n undulations ($n \geqslant 0$). The transition from $P^{(n)}$ to $P^{(n+1)}$ follows two different scenarios: in one case, the $P^{(n)}$ regime shows a succession of period doubling bifurcations culminating in chaos, from which merges the $P^{(n+1)}$ regime [3]; in a second case, there is just an intermediary region between the two regimes, where the system seems to hesitate between $P^{(n)}$ and $P^{(n+1)}$ (fig. 1a). This intermediate regime has been called $P^{(n)}-P^{(n+1)}$ hesitation [5]. Under a close scrutiny, it appears that the $P^{(n)}$ regime is rarely perfectly periodic, and fluctuations appear in the pulse period (fig. 1b). These fluctuations increase drastically when the system approaches the PQS-off or PQS-on bifurcation points.

(iii) For strong absorption, a $\bar{P}^{(n)}$ PQS regime takes place, similar to the $P^{(n)}$ one, except that the pulse shape shows a slope change after the first peak and secondary undulations with a smaller amplitude. This regime presents all the characteristics of the $P^{(n)}$-PQS, and particularly fluctuations and hesitations.

The evolution of the PQS pulses is determined by the existence of two fixed unstable points in the phase space. One of them, I_0, is the residue of the $I=0$ stable solution under the laser threshold, the second, I_+, is a precursor of the CW stable solution at high gain [7]. Concerning the PQS dynamics [5], we may schematically say that, when the I_0 point may be neglected because of his weak attraction power, type-II PQS appears and chaos may occur through a Fei-

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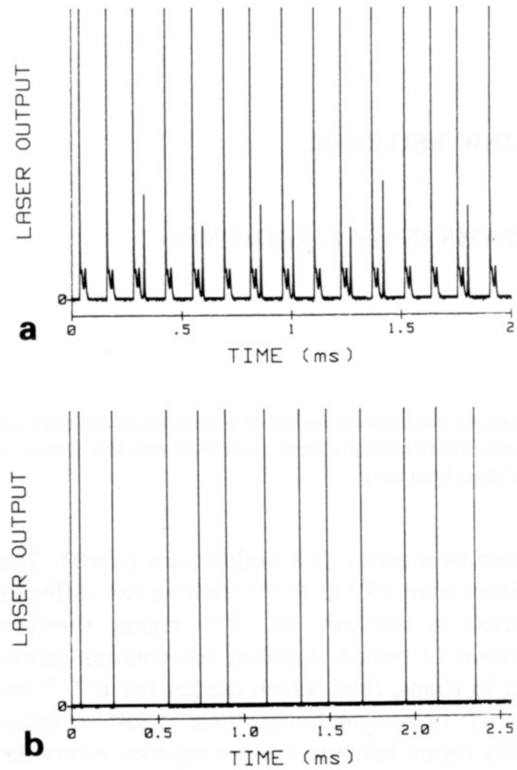

Fig. 1. (a) $P^{(2)}$–$P^{(3)}$ hesitations on the laser output power (measured in arbitrary units) for LSA operating on 10P30 CO_2 with 50 mTorr SF_6 at $A=1.017$ and laser frequency at center of cavity mode. (b) Period fluctuations in a $P^{(0)}$ regime of a 10P30 CO_2 LSA with 60 mTorr SF_6, operating at $A \approx 1$.

genbaum scenario. On the contrary, when the I_0 fixed point is attractive enough to influence the phase space trajectory, a $P^{(n)}$ or $\bar{P}^{(n)}$ regime is stabilized, and chaos tends to disappear. In an intermediate situation, when the I_0 attraction has a weak influence on the trajectory, period doubling transitions on $P^{(n)}$ pulses are observed. In this case, the dynamics remains principally dependant on I_+, and the phase space structure is very closely related to the Shil'nikov one [9], where chaos is shown to exist.

The period fluctuations and hesitations we have presented in fig. 1 in the case of pure $P^{(n)}$ regimes, have not received an interpretation. Two possible explanations have been pointed out [5].

(i) The Shil'nikov structure remains predominant, and the fluctuations and hesitations are evidence of the Shil'nikov chaos, as observed in the CO_2 laser with modulated losses [9]. The characterization of such a chaos relies principally on the variations of the return times, those on the amplitude remaining very small.

(ii) The fluctuations and hesitations have a noisy origin, because either a region of Shil'nikov chaos is not present or, if present, it has a small amplitude on the scale of the control parameters as compared to the noise level. In this case, the fluctuations and hesitations between adjacent $P^{(n)}$ or $\bar{P}^{(n)}$ regimes are connected to a stochastic mixture of the adjacent regimes in the laser operation because of the noise presence.

The tentatives to evidence a chaotic structure inside the fluctuations and hesitations have so far failed [5], and we present here a stochastic study of the PQS operation. The influence of an external added noise on PQS regimes has revealed a very high sensitivity of the pulse period on the noise level. Statistical distributions of the pulse periods as function of the applied noise have being investigated. Extrapolations of the results obtained in presence of external noise to an external noise equal to zero have showed that the internal noise in our LSA system was large enough to generate the fluctuations typically observed in PQS, and to perturb an eventual chaotic behavior in the $P^{(n)}-P^{(n+1)}$ passage. The introduction of a stochastic term within a deterministic model well describing the other characteristics of PQS, has confirmed the experimental observations.

2. Experiment

In the experiment, the LSA was constituted by a cavity containing a gain medium and a saturable absorber [10]. We used a CO_2 laser on 10P band lines with low pressure SF_6 as saturable absorber to explore easily both the $P^{(n)}$ and $\bar{P}^{(n)}$ regimes. An HgCdTe detector at the output of the laser produced a voltage proportional to the intensity I of the laser. Signal acquisition and treatment were done through a zero-crossing counter, a transient digitizer, and transfer to a micro-computer.

Three experimental parameters mainly influence the LSA operation: the cavity length, the amplifier discharge current and the absorber pressure. The relation between these parameters and the theoretical ones is not straightforward, except for the discharge current which in our case is proportional to the amplifier pump parameter A which results equal to 1 for the threshold of the laser operation without absorber [11]. Thus, to make easy the interpretation, the external noise was added to the amplifier discharge current. The noise level will be measured by the rms $(\langle \Delta A^2 \rangle)^{1/2}$ of the pump parameter. We used an home made noise generator which delivers a colored gaussian noise centered at zero frequency whose frequency bandwidth was measured on the laser output power without absorber [12].

Our first observations concerned the influence of the noise on the return time T of the PQS signals. The return time is the time taken by the trajectory to come back in the same point of the phase space, and corresponds to the pulse period when the signal is periodic. The return time was measured on the laser output temporal signal through the zero-crossing counter. In the measurements, the zero level was adjusted to be very close to the zero intensity. The statistical distribution of the return times was obtained by monitoring a thousand PQS pulses.

The statistical distributions were observed for different values of the pump parameter, corresponding to different mean return times, and for different levels of the noise, as shown on fig. 2. The addition of noise on the pump parameter broadens the distribution. The broadening is already visible for a noise

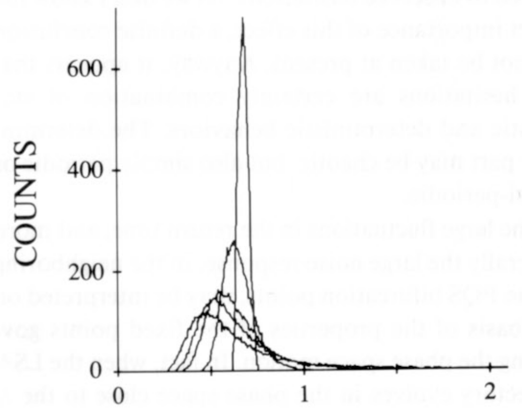

Fig. 2. Distribution of the PQS return time T at different levels of external noise. From the narrow to wide distributions: no external noise, and then respectively 0.05, 0.09, 0.12 and 0.16 r.m.s. noise level for $\langle A \rangle = 1.23$ and a 0.5 kHz noise bandwidth. 10P26 CO_2 LSA with 30 mTorr SF_6.

level of 0.01 rms, and a saturation appears for levels around 0.02 rms, as shown on fig. 3, where the hwhm ΔT of the distribution is represented as function of the noise level.

Fig. 2 shows also that the addition of noise displaces the maximum of the statistical distribution, and creates an asymmetry in the curves. These behaviors are characteristic of a system operating with a component of multiplicative noise. The fact that laser conditions exist where these phenomena are weaker indicates that the amount of applied multiplicative noise depends on the laser operation point.

The influence of the noise bandwidth on the return time statistical distribution was also tested. We present in fig. 3 the noise effect for two different cases: a noise with low frequency components in (a), and a high frequency noise in (b). A low frequency colored noise is more efficient than a broadband noise in modifying the return time distributions. However, in both cases, the LSA response to the noise remains the same, i.e. high sensitivity at low noise levels and saturation at higher levels. An examination of the return time statistical distribution without external noise permitted to estimate that the residual fluctuations corresponded to a noise of about 0.003 rms on the pump parameter. This number is considered as a good approximation of the level of internal noise in our system. Thus, the fluctuations of the return times in the $P^{(n)}$ and $\bar{P}^{(n)}$ regimes appear to result from the noise.

Our second observations concerned the influence of noise on the amplitude of the PQS signal, and more

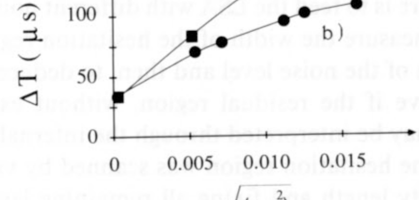

Fig. 3. Evolution of the hwhm ΔT of the return time distribution as function of the level $(\langle \Delta A^2 \rangle)^{1/2}$ of the applied external noise. Bandwidth of the noise was 0.5 kHz in (a) and 20 kHz in (b). The lines are not a fit of the experimental data, and have been drawn to help interpretation. 10P26 CO_2 LSA with 30 mTorr SF_6, at $\langle A \rangle = 1.23$

generally on the pulse shapes. The maximum information about these characteristics is derived by reconstructing the trajectory of the system in the phase space, i.e. a phase portrait. The most convenient two-dimensional phase space is the $(I, \dot{I})$ one, obtained by sending on an oscilloscope used in the X–Y mode, the intensity and its analogic derivate, as presented previously [2].

It appeared that the $P^{(n)}$ and $\bar{P}^{(n)}$ phase portraits are in general relatively insensitive to noise. Whichever the noise bandwidth is only at a level around 0.10 to 0.15 rms on the pump parameter, a general degradation of the orbit was observed, but no qualitative changes occurred. An exceptional situation arised in the proximity of transitions $P^{(n)}-P^{(n+1)}$ or $\bar{P}^{(n)}-\bar{P}^{(n+1)}$. There, the system appeared very sensitive to the noise, and a qualitative change was observed on the trajectory. For example, adding noise on a $P^{(n)}$ trajectory for a laser operating regime close to the passage to $P^{(n+1)}$ trajectories produced the apparition of the $P^{(n+1)}$ branch in the phase portrait. This behavior is similar to that appearing in the hesitation regime with the trajectory returning to the I_0 point after either n or $n+1$ loops around the saddle focus point I_+. The noise level required for this deformation of the PQS pulses depended on the system parameters values, but was around 0.01 rms on the pump parameter for operation close to an hesitation region at low n numbers. Thus, the PQS trajectories are very sensitive to noise in the proximity of $P^{(n)}-P^{(n+1)}$ or $\bar{P}^{(n)}-\bar{P}^{(n+1)}$ transitions.

This behavior, not surprising in the vicinity of a bifurcation point, permits to determine the degree of sensitivity of the system to the noise, and to deduce if the origin of hesitations is stochastic only. The procedure is to feed the LSA with different noise levels, to measure the width of the hesitation region as function of the noise level and then, to deduce from this curve if the residual region, without external noise, may be interpreted through the internal noise level. The hesitation region was scanned by varying the cavity length and fixing all remaining laser parameters. Thus, the width δ of the hesitation regime corresponds to an interval of frequency detuning. An example of width of the hesitation region versus the noise level is shown on fig. 4. This figure allows us to estimate the internal noise level required to explain the residual hesitations to be around 0.07 rms

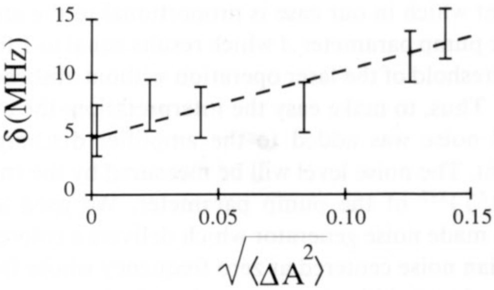

Fig. 4. Extension δ of the hesitation zone versus the applied external noise $(\langle \Delta A^2 \rangle)^{1/2}$. The dashed line is obtained by fitting linearly the data. The bandwidth of the noise was 0.5 kHz. 10P30 CO_2 LSA with 50 mTorr SF_6, at $\langle A \rangle = 1.017$ with laser frequency around center of cavity mode.

if we suppose the hesitation region varying linearly with the noise level. Depending on thé laser parameters, this value may decrease down to 0.05 rms. Now, the residual noise of our system, estimated above from the return time statistical distribution, is less than 0.01 rms. Thus, the residual internal noise appears not to be at the origin of the hesitations.

In this analysis the LSA response was supposed to be proportional to the noise level, but as seen above, the response is probably not linear, showing a saturation effect at very low noise levels. This nonlinearity may decrease the level of the internal noise required to observed hesitations. As we don't know the exact importance of this effect, a definite conclusion cannot be taken at present. Anyway, it appears that the hesitations are certainly combination of stochastic and deterministic behaviors. The deterministic part may be chaotic, but also simply periodic or quasi-periodic.

The large fluctuations in the return time, and more generally the large noise response, in the neighboring of the PQS bifurcation points, may be interpreted on the basis of the properties of the fixed points governing the phase space motion. In fact, when the LSA trajectory evolves in the phase space close to the I_0 or I_+ stable manifolds, the motion becomes very slow. The noise may perturb this slow motion and the time spent in the vicinity of those points, whence the return time. Moreover a noise at low frequency is more efficient in perturbing that LSA slow evolution as reported in fig. 3.

3. Theoretical analysis

In order to test the interpretation presented for the LSA response noise, numerical investigations have been performed on the model with three levels in the amplifier and four levels in the absorber introduced in ref. [7], which takes into account the rotational coupling in the absorber and presently describes more precisely the LSA phenomenology. This model does not include the amplifier and absorber detunings characterizing the laser frequency. Thus, a quantitative comparison with the experiments cannot yet be performed, and the model may be used only to provide a qualitative interpretation of the phenomena observed during experiments. The fluctuations have never been observed in absence of noise on the time evolution of the laser intensity obtained with this model, neither the hesitations. The $P^{(n)}-P^{(n+1)}$ transitions always occurred through period doubling cascades culminating in chaos. However, changing the parameters of the system, the intermediary region could become very narrow, and even disappear within the limits provided by the computation precision.

The addition of a stochastic term to the control parameter of the amplifier permitted to simulate the experimental addition of noise on the pump parameter. That noise acts on the laser evolution through both additive and multiplicative terms, in agreement with the experimental observations. Concerning the $P^{(n)}-P^{(n+1)}$ transition, the model with added noise presents a new scenario. In fact, for the parameters corresponding to the very narrow transition region in the deterministic model, hesitations are observed. The regime is composed of a mixture of pulses with $P^{(n)}$ and $P^{(n+1)}$ shape. Fig. 5 shows the numerical results for the $P^{(0)}$ and $P^{(1)}$ hesitation obtained for a set of parameters corresponding to the LSA experimental conditions. By using the parameters introduced in refs. [4] and [11], the following rates, measured in units of the cavity damping rate 2κ, have been used: for the upper and lower states of the gain medium, $R_{10}=3.4\times10^{-3}$ and $R_{20}=2.7\times10^{-2}$ respectively; for the absorber vibrational rate $\gamma=1.5\times10^{-3}$, rotational rate $\bar{\gamma}_r=0.73$ and rotational partition function $Z_r=2\times10^{-3}$. The ratio between the cross sections of the gain stimulated emission and that of the absorption in the absorbing medium is $B_g/B_a=1/80$, the gain parameter $A=1.14$

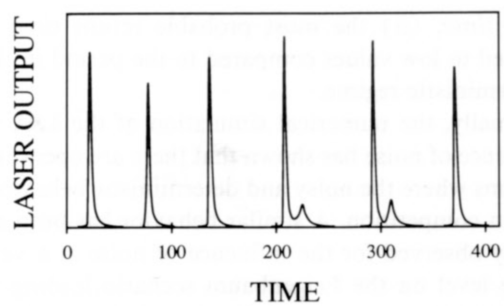

Fig. 5. $P^{(0)}-P^{(1)}$ hesitations in the LSA model in presence of noise. The time is expressed in reduced units rapported to the cavity losses and the intensity in arbitrary units.

and the absorber parameter $\bar{A}=0.17$. In fig. 5, a colored noise was applied to the gain parameter with rms $(\langle\delta A^2\rangle)^{1/2}=0.18$ and correlation time $\tau_c=2\times10^3/2\kappa$, and the spontaneous emission in the gain medium was described through an additive term with $(\langle\delta I^2\rangle)^{1/2}=3\times10^{-8}\ I$ on the I intensity. The results of fig. 5 from the numerical analysis demonstrate that the noise presence leads to hesitations.

Another difference between the deterministic and stochastic evolutions concerns the fluctuations that, absent in the deterministic case, appear when noise is added. An evaluation of the fluctuation presence is obtained through the construction of statistical histograms of the return time (see fig. 6), as those recorded in the experiments. Such histograms require a large amount of computational time if a reasonable accuracy in the resolution is envisaged. That explains the few points presented in the example of fig. 6. Anyway, fig. 6 reports two features observed experimentally: (i) the distribution presents an asymmetry with a long tail for large values of the re-

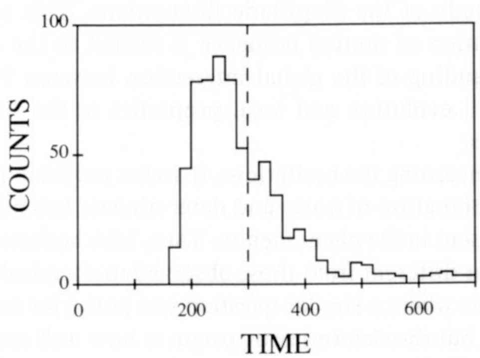

Fig. 6. Histogram of the return times in the LSA model. The time is expressed in reduced units as in fig. 5. The dashed line represents the regime period in the deterministic model in abscence of noise.

turn time; (ii) the most probable return time is shifted to low values compared to the period in the deterministic regime.

Finally, the numerical simulation of the LSA in presence of noise has shown that there are operating regions where the noisy and deterministic behaviors are in competition. A similar behavior has been already observed for the influence of noise at a very large level on the Feigenbaum scenario leading to chaos [12]: there, a destabilization of the chaotic regime may occur, and the system may jump to a regime with periodic orbits. Now, in the LSA operation, the width of the region in the phase diagram where period doublings and chaos take place is very narrow, so that whichever external or internal noise, the level is always very large as compared to the width of the region. Thus also in the Shil'nikov situation at large noise levels, competition of noisy and deterministic behaviors occurs.

In conclusion, we have related the fluctuations in the return time to the influence of internal or external added noise. The experimental and theoretical investigations have pointed out the strong role played by the multiplicative component of the noise. The noise role in the LSA phase space evolution is connected to the lethargic times spent around the two LSA fixed points. During these lethargic times, the low frequency components of the noisy perturbation may modify the LSA phase space trajectory. The influence that the actual position of the trajectory in the proximity of one fixed point has on its position around the other one, has not been studied here. Information about this influence may be derived from the study of the amplitude fluctuations. This consideration of mutual influence is related to the understanding of the global interaction between PQS overall evolution and local properties at the fixed points.

Concerning the hesitations, it seems probable that a combination of noisy and deterministic behaviors is present in the phenomenon. Thus, LSA hesitations appear different from those observed in chemical reactions where a similar question was posed for some time, but the deterministic origin is now well established [13].

Several questions are still open and will require future investigations. For instance, a more precise characterization of a regime corresponding to a competition of noisy and deterministic behaviors is required, even if the strong perturbation of the periodic or quasi-periodic deterministic part by the noise could make very difficult such investigation. In the future, a more complete experimental and theoretical investigation of the hesitations may be performed only if this phenomenon will take place on a wider region of the control parameters. In fact, even in our apparatus, where internal noise has been reduced to an extremely low level, the deterministic evolution of the hesitations cannot be checked.

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