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Spatio-temporal dynamics of a weakly multimode CO_2 laser

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The behavior of a slightly asymmetric weakly multimode CO_2 laser is studied experimentally and compared with that of perfectly cylindrically symmetrical lasers. It is found that in most cases, the system symmetry breaking destroys the vortices, although it appears that for a very small loss of symmetry, the behavior of the laser remains symmetrical.

Interest in the spatial dynamics of nonlinear optical systems has particularly increased since Coullet et al. demonstrated the possible existence of defect-mediated-turbulence in such systems [1,2]. These studies have recently concentrated around a series of theoretical [2,3] and experimental [3–6] works to characterize these defects, sometimes called phase singularities or optical vortices [7] by analogy with the vortex structures appearing in hydrodynamics. Two approaches have been followed: (i) the first one considers directly the strongly multimode case, which requires at the theoretical level a direct resolution of the partial differential equations modelling the system [2]; (ii) the second one consists in analysing progressively the dynamics of the system as the number of modes is increased. Theoretically, this second problem has been taken up by Lugiato et al. through the modal decomposition for a cylindrically symmetrical optically pumped ring laser and the results have been experimentally checked on a Na_2 laser by Weiss et al. [3]. They showed in particular that as soon as two modes are present in the cavity, a vortex may appear.

It is interesting to check experimentally the robustness of these predictions for different conditions of symmetry. We hence report the analysis of the behavior of a Fabry–Pérot CO_2 laser without any compensation of the astigmatism introduced by the optical elements. Using a Fabry–Pérot instead of a ring cavity increases the symmetry, but different experimental factors reduce this symmetry and lift the degeneracy of modes. They include electrical instead of optical pumping, but the main cause of symmetry loss is the astigmatism introduced in particular by the Brewster windows. We show that this loss of symmetry acts on the modal solutions predicted in ref. [3] in three different ways: (i) some modes completely disappear from the phase diagram of the system, (ii) some are left unchanged in their time average but acquire temporal dynamics and (iii) the spatial pattern of other modes, in particular those with cylindrical symmetry, is modified, even in its time average. All these effects depend strongly on the frequency splitting between quasi-degenerate modes. Mode frequency locking, that occurs for small splittings, tends to restore the symmetry.

The CO_2 laser used here consists of a 2 m long Fabry–Pérot cavity limited by two plane mirrors, one of them transmitting 5% of the light. The 0.8 m long amplifier tube is closed by two ZnSe Brewster windows. Adjusting the position of an intracavity lens allows the stability of the cavity, and consequently the Fresnel number to be varied, following a method introduced in ref. [5]. The results presented here correspond to a situation where the Fresnel number is typically of the order of 4, obtained with a cavity aperture diameter of 1.25 cm and a lens of focal

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length 10 cm located between 10 and 10.5 cm from one of the plane mirrors.

The modes of such a cavity can be expanded in different bases according to the symmetry of the system. For a cylindrically symmetrical system, where the Laguerre–Gauss basis set is best suited, a mode is noted A_{pli}, as introduced in ref. [3], where p and l are respectively the radial and angular indices, and i can take the values 1 and 2 thus differentiating two similar modes shifted by $\pi/2l$. In such a basis, the mode frequency depends only on the value of the number $q=2p+l$. Consequently, the modes are gathered in families of $q+1$ modes having the same frequency.

These results are valid for a cylindrically symmetrical cavity. In our case, the presence of Brewster windows and, to a certain extent, the lens, induces astigmatism in the cavity. This astigmatism (i) disturbs the cavity losses in the transverse direction, so that a mode with an energy mainly distributed along one of the Brewster windows axis will have more gain than the perpendicular mode, and (ii) lifts the degeneracy of modes belonging to the same family.

In the present experimental conditions, the transverse mode spacing is about 20 MHz for a free spectral range of 80 MHz, and the width of the net gain curve is smaller than the transverse intermode spacing. As a consequence, when the cavity detuning is varied, successive families of modes are selectively excited, except for small ranges where two families may overlap. Inside a family, the cavity detuning remains a control parameter together with the diameter of an iris located inside the cavity, and allows jumping between dynamical modes. The frequency difference $\Delta\nu_f$ between modes of the same family due to the lifting of degeneracy varies from less than 400 kHz up to 1.2 MHz, depending on cavity alignment and gain.

We will now examine the behavior of the laser for the first two families $q=1$ and $q=2$. The $q=0$ family consists of only the A_{00} mode, which corresponds to the TEM_{00} Hermite–Gauss mode. Its symmetry is not affected by that of the system and so is not extensively dealt with.

The q=1 family

This family is composed of the two Laguerre–Gauss modes denoted A_{011} and A_{012}. They correspond to the TEM_{01} and TEM_{10} Hermite–Gauss modes. Brambilla et al. showed that within this family, the only stable laser modes are the doughnuts B_{011} and B_{012} [3], which are a linear combination of the two preceding ones. Their intensity patterns are identical and correspond to a ring, while their field configurations vary with the angular coordinate ϕ as $\exp(\pm il\phi)$, and so the cylindrical symmetry is lost. Brambilla et al. predicted the stability of these two modes in the same domain, so that there is bistability between them. This was experimentally observed by Tamm et al. [6].

Figure 1 shows the typical pattern observed in our laser when the cavity length is tuned so that the $q=1$ family is selected with $\Delta\nu_f$ larger than 400 kHz. Digital processing of this data has very much increased the contrast between the regions of the ring with a low intensity maximum and a high intensity maximum. The effective difference between these two values is 50%, and originates from the astigmatism of the cavity. Figure 1 shows the time averaged intensity of the transverse pattern, but we have checked with a fast detection that this pattern is not stationary. It evolves periodically at the frequency differ-

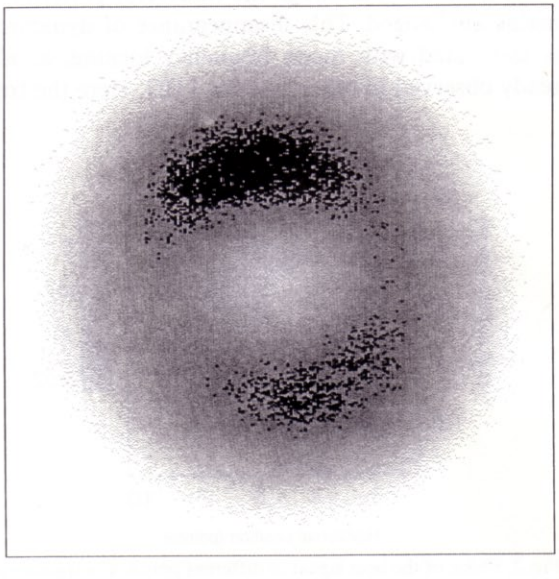

Fig. 1. High contrast video record of the transverse section output beam of the laser when the TEM_{01} and TEM_{10} modes are present in the gain profile. The asymmetry of the pattern originates in the symmetry breaking of the system due to the two-mode frequency degeneracy lift. The effective intensity difference between the two modes is of the order of 50%.

ence between the TEM_{01} and TEM_{10} modes. To check if this modulation corresponds to global dynamics of the pattern or to an antiphase behavior of the two modes, we have measured the relative phase of the modulation in more than 100 different points of the transverse plane. The experimental method consists of recording simultaneously the signal at two different points of this plane. One of these points remains unchanged during the whole experiment, and is used as a reference to compare the relative phase of the points recorded on the other channel and distributed on a grid covering the whole pattern. The histogram of the resulting values clearly shows two groups with opposite phase in spite of a large dispersion of the results (standard deviation of the order of $\pi/10$), originating essentially from the difficulty in measuring precisely the phase of small amplitude signals. The distribution of relative phases in the transverse plane (fig. 2) leads to the conclusion that the two modes pulse in antiphase, i.e. one of the mode is at its maximum intensity when the other one is at zero intensity. This is the simplest case of the periodic alternance defined for spatio-temporal dynamics [8,9].

We have observed that the laser output may become stationary, although the transverse pattern remains unchanged. This disappearance of dynamics is associated with mode frequency locking, as already observed in other systems [10]. Here the frequencies lock as soon as in some conditions $\Delta\nu_f$ decreases to less than 400 kHz. In this case, the system tends to return to a higher symmetry than the real one and the pattern is very similar to the one observed in absence of asymmetry. It is also possible to induce this symmetric behavior in a parameter range where the antiphase motion is observed. In this case, there is bistability between the two regimes (locked and antiphase), but any perturbation always brings back the antiphase behavior.

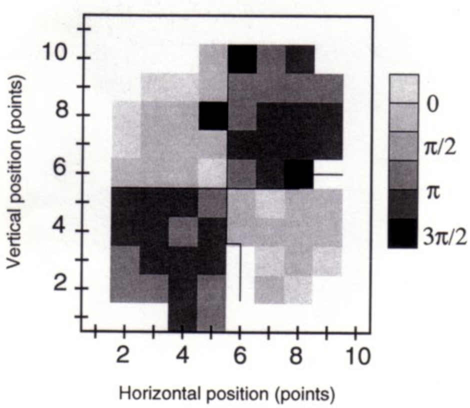

Fig. 2. Phase of the beat signal in different points of a transverse pattern similar to the one of fig. 1. The phase origin has been arbitrarily chosen at the point of coordinate (2,6). The white squares correspond to points where the modulation amplitude is too small to measure the phase. Note that three points are in phase quadrature with the other ones. This originates probably in a detection artefact.

It is interesting to analyse this very simple situation from the point of view of turbulence. A necessary condition for turbulence as introduced in ref. [1] is the existence of defects. In our case, the experimental visualization of such an object is not easy, as no fast infrared cameras are available and there is no reference beam for monitoring phases through interference patterns. However, in the situation where the two modes beat in antiphase, the resulting pattern is a simple superimposition of the two modes, without any phase combination, and so there is no phase singularity. As a consequence, it appears that whereas a perfectly symmetric system could be turbulent because of associated defects, in fact a loss of symmetry destroys these vortices so that turbulent behavior is no longer possible. In the case of frequency locking, the similarity of the resulting pattern to those predicted and observed in ref. [3] suggests the existence of a phase singularity in the center of the mode. This means that a sort of memory exists, so that moving the system slightly away from the strict symmetry has no consequence on the behavior. This memory is also responsible for bistability when the parameters are continuously changed from a region where the system is quasi-symmetric to a region where it is completely asymmetric.

The q=2 family

In this case, three modes compete, namely the A_{10} (a ring with a central point) and the two A_{02} ones (four-spot mode). Brambilla et al. have predicted in a class C laser the stability of five modes, the A_{10}, the two doughnuts B_{02}, the four-hole and the two-hole ones. We are not able in our system to observe the doughnut modes, they seem not to survive to the loss of symmetry of the system. We have experimentally observed all the other predicted modes (fig. 3), in particular the four-hole (fig. 3b) and the two-hole (fig. 3c) modes, that display respectively four and

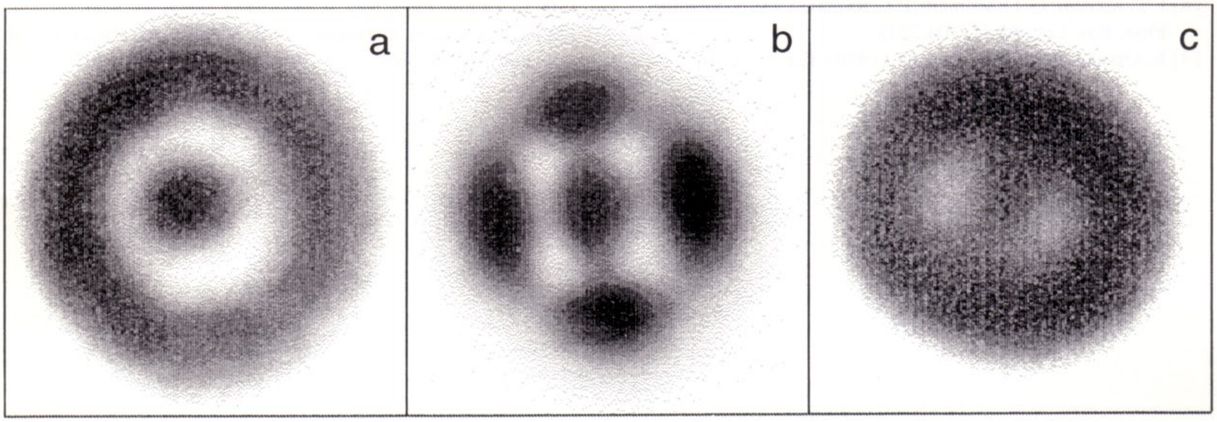

Fig. 3. Patterns observed at the output of the laser when the $q=2$ mode family is present in the cavity. From (a) to (c), the so-called A_{10}, four-hole and two-hole patterns are shown.

two vortices at the points of zero intensity in theoretical models. However, in the experiment, these modes show dynamics which includes now two frequencies corresponding to the beating between the three modes, together with their combination tones. It appears clearly that the relative level of these various spectral components has a spatial dependence. This complexity does not allow us to perform a phase map similar to the one carried out for the $q=1$ case. However, the spectrum spatial dependence involves that the whole pattern is composed of different modes whose temporal characteristics are not the same. This is not consistent with the hypothesis of a rigid dynamical pattern preserving the properties of the stationary one, and in particular the vortices. Although a more precise study is necessary, we can reasonably suppose that the vortices are also destroyed by asymmetry for the $q=2$ family.

As a conclusion, we have shown that in a weakly multimode laser, a small symmetry breaking does not alter significantly the time average transverse pattern. In fact, we were able to observe all the patterns predicted in the perfectly symmetric case [3] for the three first mode families, except the $q=2$ doughnuts. However, this agreement is only superficial, as these patterns acquire temporal dynamics originating from the degeneracy lift of the cavity mode frequencies. This spatio-temporal dynamics appears as periodic alternance of modes and results in a destroying of the defects and so of the possibility to observe turbulence. It appears therefore that a higher symmetry makes the appearance of turbulence easier. Finally, we observe that in the case $q=1$, when the system approaches the perfectly symmetric case, the phenomenon of mode frequency locking tends to restore the spatial symmetry of the pattern, and in particular the defects.

Note added in proof: We should like to make reference to a very recently published paper entitled, Spatio-temporal dynamics of lasers in the presence of an imperfect O(2) symmetry, by E.T. d'Angelo et al. [11].

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